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A NEW UNIVARIATE MAX POLICY OF RECRUITMENT IN A TWO GRADED MANPOWER SYSTEM WITH DIFFERENT EPOCHS FOR DECISIONS AND EXITS

G. Ishwarya*
A. Srinivasan**

Abstract

In this paper, the problem of time to recruitment is studied for a two graded manpower system by employing a new univariate MAX policy of recruitment in which exits take place due to policy decisions. Assuming that the policy decisions and exits occur at different epochs, three mathematical models are constructed based on shock model approach and the variance of time to recruitment is obtained when the inter-exit times form an ordinary renewal process according as inter-policy decision times form a sequence of exchangeable and constantly correlated exponential random variables or a geometric process or an order statistics. The analytical results in closed form are derived for all three models by assuming specific distribution to loss of manpower.

Keywords:

Two graded manpower system;
Decision and exit epochs;
geometric process;
ordinary renewal process;
order statistics;
correlated and exchangeable random variables;
New Univariate MAX policy of recruitment; Mean and variance of time to recruitment.

Author correspondence:

G. Ishwarya,
Assistant Professor, Department of Mathematics,
KCG College of Technology, Karapakkam, Chennai - 600097

1. Introduction

Attrition is a common phenomenon in many organizations when policy decisions such as revising sales target, emoluments etc. are announced and this leads to the instantaneous depletion of manpower. Recruitment on every occasion of depletion of manpower is not advisable since every recruitment involves cost. As the depletion of manpower is unpredictable, a suitable recruitment policy has to be designed to overcome this loss. One univariate recruitment policy which is often used in the literature is based on shock model approach for replacement of system in reliability theory. In this policy, known as univariate CUM policy of recruitment, the cumulative loss of manpower is permitted till it reaches a level, called the breakdown threshold and when this cumulative loss exceeds the threshold, recruitment is carried out. In [1, 5] the authors have discussed several manpower planning models using Markovian and renewal theoretic approach. In [16] this problem is studied for the first time using this policy. In [17], the authors have studied the problem of time to recruitment for a single grade manpower system and obtained the variance of

* Assistant Professor, Department of Mathematics, KCG College of Technology, Chennai – 97.

** *Professor Emeritus, PG & Research Department of Mathematics, Bishop Heber College,Trichy-17*

time to recruitment when the loss of manpower forms a sequence of independent and identically distributed random variables, the inter-decision times are independent and identically distributed exponential random variables and the mandatory threshold for the loss of manpower is assumed as constant using univariate MAX policy of recruitment. This work has been extended by the same authors in [18] for random threshold. In all the above cited models, it is considered that the loss of manpower is instantaneous when the decision is taken. This assumption is not realistic as the actual attrition will take place only at exit points which may or may not coincide with decisions points. The concept of non-instantaneous loss of manpower in decision epochs has been introduced for the first time in [2] for a single grade manpower system and the performance measures are obtained for the same when the loss of manpower are exponential random variables, inter-decision times are independent and identically distributed exponential random variables and for exponential threshold. In [3], the authors have extended the research work in [2] using Indicatory technique method. Again the work has been extended in [4] by the same author for Univariate MAX policy of recruitment according as inter-decision times form exchangeable and constantly correlated exponential random variables or a geometric process or an order statistics using the above cited method. In [7, 8, 9 & 10], the authors have extended the work in [2] for a two graded manpower system and obtained the performance measures according as the inter decision times are independent and identically distributed exponential random variables or exchangeable and constantly correlated exponential random variables using the above cited techniques. In [11 & 12], the work has been extended for geometric process of inter-decision times and for an order statistics. The same authors have extended the above cited works in [13] for Univariate MAX policy of recruitment. In this paper, the research work in [13] is studied for a new Univariate MAX policy of recruitment which is stated in the model description.

2. Model description and Analysis

Consider an organization with two grades (grade-1and grade-2) taking policy decisions at random epochs in (0, ∞) and at every decision making epoch a random number of persons quit the organization. There is an associated loss of manpower, if a person quits. It is assumed that the epochs for decisions and exits are different and the loss of manpower is linear and cumulative. For i=1,2,3…, let X_{i1} & X_{i2} be independent and identically distributed exponential random variables representing the amount of depletion of manpower (loss of man hours) in grade-1 & 2 at the ith exit point with probability distribution $M_1(.)\,\&\,M_2(.)$, density function $m_1(.)\,\&\,m_2(.)$ and mean $\frac{1}{\lambda_1}\,\&\,\frac{1}{\lambda_2}$ (λ_1, λ_2 >0) respectively. Let $Z_{i1}\,\&\,Z_{i2}$ be the maximum loss of manpower in the first 'i' decisions for grade 1 & 2 respectively. Let U_i be the continuous random variable representing the time between (i-1)th and ith policy decisions. Let $F(.)$ and $f(.)$ be the distribution function and density function of U_1 respectively with mean $\frac{1}{\theta}$ (θ >0). Let $f^*(.)$ be the Laplace transform of $f(.)$. Let $F_n(.)$ and $f_n(.)$ be the distribution and density function of the random variable $U_1+U_2+...+U_n$ respectively and $f_n^*(.)$ be the Laplace transform of $f_n(.)$. Let R_i be the time between (i-1)th and ith exits. It is assumed that R_i's are independent and identically distributed random variables with distribution function $G(.)$ and density function $g(.)$. Let D_{i+1} be the waiting time up to (i+1)th exit. Let $E(R)$ and $V(R)$ be the mean and variance of the inter-exit times respectively. Let Y be the breakdown threshold which is taken as constant for the loss of man hours in the organization. Let q (q≠0) be the probability that every policy decision produces an attrition. Let T be a continuous random variable denoting the time for recruitment with mean $E(T)$ and variance $V(T)$.

The new univariate MAX policy of recruitment employed in this paper is stated as follows: **Recruitment is done whenever the maximum of the maximum total number of exits in grade – 1 and the maximum total number of exits in grade – 2 due to policy decisions exceeds the constant threshold Y**.

We now obtain the variance of time to recruitment. By the probabilistic arguments, the time to recruitment can be written as

$$T=\sum_{i=0}^{\infty} D_{i+1}\, I\left[\max(Z_{i1},Z_{i2})\le Y<\max(Z_{(i+1)1},Z_{(i+1)2})\right]$$

and

$$E(T)=E(R)\sum_{i=0}^{\infty}(i+1)\,P\begin{bmatrix}\max(Z_{i1},Z_{i2})\le Y\\ <\max(Z_{(i+1)1},Z_{(i+1)2})\end{bmatrix} \quad \text{.........(1)}$$

Similarly $T^2=\sum_{i=0}^{\infty}D_{i+1}{}^2\;I\left[\max(Z_{i1},Z_{i2})\le Y<\max(Z_{(i+1)1},Z_{(i+1)2})\right]$

and

$$E(T^2)=V(R)\sum_{i=0}^{\infty}(i+1)\,P\begin{bmatrix}\max(Z_{i1},Z_{i2})\le Y\\ <\max(Z_{(i+1)1},Z_{(i+1)2})\end{bmatrix}$$
$$+[E(R)]^2\sum_{i=0}^{\infty}(i+1)^2\,P\begin{bmatrix}\max(Z_{i1},Z_{i2})\le Y\\ <\max(Z_{(i+1)1},Z_{(i+1)2})\end{bmatrix} \quad \text{..........(2)}$$

We now compute, $P\left[\max(Z_{i1},Z_{i2})\le Y<\max(Z_{(i+1)1},Z_{(i+1)2})\right]$

Let $\max(Z_{i1},Z_{i2})=\overline{\overline{Z_i}}$ and $\max(Z_{(i+1)1},Z_{(i+1)2})=\overline{\overline{Z_{i+1}}}$

$$P\begin{bmatrix}\max(Z_{i1},Z_{i2})\le Y\\ <\max(Z_{(i+1)1},Z_{(i+1)2})\end{bmatrix}=P\left(\overline{\overline{Z_i}}\le Y<\overline{\overline{Z_{i+1}}}\right)$$
$$=P\left\{\left(\overline{\overline{Z_i}}\le Y\right)\cap\begin{bmatrix}\left\{\left(X_{(i+1)1}>Y\right)\,and\,\left(X_{(i+1)2}\le Y\right)\right\}\\ \cup\left\{\left(X_{(i+1)1}\le Y\right)\,and\,\left(X_{(i+1)2}>Y\right)\right\}\\ \cup\left\{\left(X_{(i+1)1}>Y\right)\,and\,\left(X_{(i+1)2}>Y\right)\right\}\end{bmatrix}\right\}$$
$$=\left[M_1(Y)M_2(Y)\right]^i-\left[M_1(Y)M_2(Y)\right]^{i+1} \quad \text{..........(3)}$$

Substitute equation (3) in equations (1) and (2), we get

$$E(T)=E(R)\sum_{i=0}^{\infty}(i+1)\left\{\left[M_1(Y)M_2(Y)\right]^i-\left[M_1(Y)M_2(Y)\right]^{i+1}\right\}$$
$$=E(R)\left[1-M_1(Y)M_2(Y)\right]^{-1}$$

i.e., $E(T)=\dfrac{E(R)}{\left[1-M_1(Y)M_2(Y)\right]}$ ....(4)

and

$$E(T^2)=\frac{V(R)}{\left[1-M_1(Y)M_2(Y)\right]}+\left[E(R)\right]^2\left\{\begin{matrix}\dfrac{2M_1(Y)M_2(Y)}{\left[1-M_1(Y)M_2(Y)\right]^2}\\ +\dfrac{1}{\left[1-M_1(Y)M_2(Y)\right]}\end{matrix}\right\} \quad \text{..........(5)}$$

where $M_1(Y)=1-e^{-\lambda_1 Y}$ and $M_2(Y)=1-e^{-\lambda_2 Y}$ ....(6)

We now obtain the explicit expression for E(T) and E(T^2) for different forms of U_i's.

<u>Model – I</u> : U_i's **are exchangeable and constantly correlated exponential random variables with mean** $\frac{1}{\theta}$**. Assume that "** R **" be the correlation between** U_i **and** U_j**,** $i\neq j$**.**

It can be proved that

$$G(x)=\sum_{n=1}^{\infty}q(1-q)^{n-1}F_n(x) \quad \text{..........(7)}$$

The Laplace Stieltjes transform of $F_k(.)$ is give by $\dfrac{(1-R)(A(s))^k}{(1-R)+kR(1-A(s))}$, ……..…(8)

where $A(s)=\dfrac{1}{1+bs}$; $b=\dfrac{1-R}{\theta}$

Using equations (7) and (8), we can prove that

$$E(R)=\frac{b}{(1-R)q}$$

and $V(R)=\dfrac{1}{q^2}\left(\dfrac{b}{1-R}\right)^2\left[1+2R^2(1-q)\right]$ ……….(9)

From equations (4), (5), (6) & (9), we have

$$E(T)=\frac{b}{(1-R)q\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]} \quad \text{……..(10)}$$

and

$$E(T^2)=\frac{1}{q^2}\left(\frac{b}{1-R}\right)^2\frac{\left[1+2R^2(1-q)\right]}{\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]}+\frac{b^2}{(1-R)^2q^2}\left\{\frac{2\left[1-e^{-\lambda_1 Y}-e^{-\lambda_2 Y}+e^{-(\lambda_1+\lambda_2)Y}\right]}{\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]^2}+\frac{1}{\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]}\right\}$$

……….(11)

It is known that

$V(T)=E(T^2)-\left[E(T)\right]^2$. ………(12)

Equation (12) together with equations (10) & (11) will give V(T) for Model – I.

<u>Model – II</u>: U_i's form a geometric process of independent random variables with rate 'a' (a>0). Assume that $F(.)$ and $f(.)$ be the distribution function and density function of U_1 respectively with mean $\dfrac{1}{\theta}$ (θ>0) and $F_n(t)=P\left[U_1+U_2+...+U_n\le t\right]$.

It is proved that $f_n^*(s)=\prod_{i=1}^{n}f^*\left(\dfrac{s}{a^{i-1}}\right)$ ………(13)

Using equations (7) and (13), we can prove that

$E(R)=\dfrac{a}{\theta(a-1+q)}$ and $V(R)=\left(\dfrac{a}{\theta(a-1+q)}\right)^2\left[\dfrac{a^2+(2a-1)(q-1)}{a^2-1+q}\right]$ ……….(14)

Use equation (14) in equations (4) and (5) together with (6), we get

$$E(T)=\frac{a}{\theta(a-1+q)\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]} \quad \text{…..…(15)}$$

and

$$E(T^2)=\left(\frac{a}{\theta(a-1+q)}\right)^2\left[\frac{a^2+(2a-1)(q-1)}{(a^2-1+q)\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]}\right]+\left(\frac{a^2}{\theta^2(a-1+q)^2}\right)\left\{\frac{2\left[1-e^{-\lambda_1 Y}-e^{-\lambda_2 Y}+e^{-(\lambda_1+\lambda_2)Y}\right]}{\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]^2}+\frac{1}{\left[e^{-\lambda_1 Y}+e^{-\lambda_2 Y}-e^{-(\lambda_1+\lambda_2)Y}\right]}\right\}$$

………(16)

Equation (12) together with equations (15) & (16) will give V(T) for Model – II.

Model – III: U_i**'s are independent and identically distributed exponential random variables which form an order statistics. Assume that** $F_{U_{(j)}}(.)$ **and** $f_{U_{(j)}}(.)$ **be the distribution and the probability density function of the** j^{th} **order statistics (j=1,2,…,n) selected from the sample of size 'n' from the population** $\{U_i\}_{i=1}^{\infty}$**.**

The density function of $U_{(j)}$ is given by

$$f_{U_{(j)}}(x) = \frac{n!}{(n-j)!(j-1)!}[F(x)]^{j-1}[1-F(x)]^{n-j} f(x) \qquad \text{………(17)}$$

Case (i): (j=1) $f(t) = f_{U_{(1)}}(t)$

In this case, $f(t) = n\theta\left(e^{-\theta x}\right)^n$ (by (17)) **………..(18)**

Using equations (7) and (18) it can be proved that,

$$E(R) = \frac{1}{n\theta q} \text{ and } V(R) = \frac{1}{n^2\theta^2 q^2} \qquad \text{……….(19)}$$

From equations (4), (5), (6) & (19), we have

$$E(T) = \frac{1}{(n\theta q)\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]} \qquad \text{……….(20)}$$

and

$$E(T^2) = \left(\frac{1}{n\theta q}\right)^2 \frac{1}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]} + \left(\frac{1}{n^2\theta^2 q^2}\right)\left\{\frac{2\left[1-e^{-\lambda_1 Y} - e^{-\lambda_2 Y} + e^{-(\lambda_1+\lambda_2)Y}\right]}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]^2} + \frac{1}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]}\right\} \qquad \text{………..(21)}$$

Equation (12) together with equations (20) & (21) will give V(T) for case(i) of Model – III.

Case (ii): (j=n) $f(t) = f_{U_{(n)}}(t)$

In this case, $f(t) = n[F(x)]^{n-1} f(x) = n\theta e^{-\theta x}\left(1-e^{-\theta x}\right)^{n-1}$ (by (17)) **…….(22)**

Using (7) and (22), we can also prove that

$$E(R) = \frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right),\ E(R^2) = \frac{1}{\theta^2 q}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right) + \left(\frac{2-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2 \text{ and}$$

$$V(R) = \frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right) + \left(\frac{1-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2 \qquad \text{……..(23)}$$

From equations (4), (5), (6) & (23), we have

$$E(T) = \left(\frac{1}{\theta q}\left(\sum_{j=1}^{n}\frac{1}{j}\right)\right)\frac{1}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]} \qquad \text{……….(24)}$$

and

$$E(T^2) = \frac{\left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j^2}\right) + \left(\frac{1-q}{\theta^2 q^2}\right)\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]} + \left(\frac{1}{\theta^2 q^2}\left(\sum_{j=1}^{n}\frac{1}{j}\right)^2\right)\left\{\frac{2\left[1-e^{-\lambda_1 Y} - e^{-\lambda_2 Y} + e^{-(\lambda_1+\lambda_2)Y}\right]}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]^2} + \frac{1}{\left[e^{-\lambda_1 Y} + e^{-\lambda_2 Y} - e^{-(\lambda_1+\lambda_2)Y}\right]}\right\} \qquad \text{…(25)}$$

Equation (12) together with equations (24) & (25) will give V(T) for case(ii) of Model – III.

Note:

If U_i's are independent and identically distributed exponential random variables with mean $\frac{1}{\theta}$, then E(T) and V(T) are deduced from Models – I, II and III by putting "R=0", "a=1" and "n=1" respectively.

3. Conclusion

The models discussed in this paper are found to be more realistic and new for a two graded manpower system in the context of considering (i) separate points (exit points) on the time axis for attrition, thereby removing a severe limitation on instantaneous attrition at decision epochs (ii) associating a probability for any decision to have exit points and (iii) employing a **new max policy** for recruitment. From the organization's point of view, our models are more suitable than the corresponding models with instantaneous attrition at decision epochs, as the provision of exit points at which attrition actually takes place, postpone the time to recruitment.

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